Expectations and variances

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Lecture 9

Learning objectives

After these lectures, we will be able to:

- Define and explain with examples what expectations and variances are.
- Calculate the expectation and variance of discrete and continuous random variables.
- Calculate the expectation and variance of functions of discrete and continuous random variables.
- Recall and use basic properties of expectations and variances.

Motivation: Printer lifetime

A printer has a working lifetime that is exponentially distributed with rate $\lambda = 1$ broken printer every 3 years. In English, we typically replace the printer every 3 years. Assume the company decides to change the printer every 2 years (if it hasn't broken down) or when it breaks down (whichever happens first). What is the expected time the company keeps a printer?

Expectation

When describing a random variable and its probability distribution, we sometimes are interested in answering a simple question "what should I expect?" Seeing as a random variable is inherently, well, random, expectations are important and they reveal a "center" of the probability distribution.

Definition 1 (Expectation) *With the term expectation (sometimes we use the term mean or expected value), we imply a measure of the center of the probability distribution. Intuitively, we may think of the expected value of a random variable X as an "average" of the values that X is allowed to take weighted by their respective probabilities.*

This definition is very open-ended, so we provide more specific definitions for discrete and continuous random variables in the next subsections.

Discrete random variables

Definition 2 (Expectation of a discrete random variable) *Let X be a numerically-valued discrete random variable with sample space S and probability mass function $p(x)$. Then, the expected value of X is written as $E[X]$ and is calculated as:*

$$E[X] = \sum_{x \in S} x \cdot p(x).$$

The expected value is commonly referred to as the mean *and is also written as μ.*

An unfair die

Consider an "unfair" die with sample space $S = \{1, 2, 3, 4, 5, 6\}$ and $p(1) = 1/3, p(2) = 1/6, p(3) = 1/6, p(4) = 1/6, p(5) = 1/12, p(6) = 1/12$. What is the expected value?

- $E[X] = 1 \cdot \frac{1}{3} + 2 \cdot \frac{1}{6} + 3 \cdot \frac{1}{6} + 4 \cdot \frac{1}{6} + 5 \cdot \frac{1}{12} + 6 \cdot \frac{1}{12} = \frac{33}{12} = 2.75.$

Note how the value we expect can never actually happen, as the die can only take the values of 1, 2, 3, 4, 5, or 6!

What about for a fair die, where each side (1, 2, 3, 4, 5, or 6) are equally probable? What is the expectation for this die?

Continuous random variables

Definition 3 (Expectation of a continuous random variable) *Let X be a numerically-valued real random variable defined over $(-\infty, +\infty)$ and probability density function $f(x)$. Then, the expected value of X is written as $E[X]$ and is calculated as:*

$$E[X] = \int_{-\infty}^{+\infty} x \cdot f(x) dx.$$

The expected value is commonly referred to as the mean *and is also written as μ.*

A rating system

A company is rating their employees with a system that assigns a score between 1 and 5. We assume the score is continuous (that is, a score of 4, 4.2, and 4.31478 are all valid) and the probability with which score x appears is given by pdf $f(x) = \frac{3}{124} \cdot x^2$, for $1 \leq x \leq 5$. What is the expected rating score of a random employee?

- $E[X] = \int_1^5 x \cdot f(x)dx = \frac{3}{124} \int_1^5 x^3 dx = \frac{117}{31} = 3.77.$

Good employees are the ones that receive a rating score of 4 or above. Their scores are distributed with a slightly different distribution: they follow pdf $f(x) = \frac{2}{9} \cdot x$ for $4 \leq x \leq 5$. What is the expected rating score of a good employee?

Properties of the expectation

The expectation satisfies the following properties

1. Let α be a real number and X be a random variable. Then:

$$E[\alpha \cdot X] = \alpha \cdot E[X].$$

2. Let X, Y be two random variables. Then:

$$E[X + Y] = E[X] + E[Y].$$

   - This generalizes to as many random variables as you would want to. In essence, we have for n random variables $X_1, X_2, \ldots, X_n$:

$$E\left[\sum_{i=1}^{n} X_i\right] = \sum_{i=1}^{n} E[X_i].$$

3. Combining 1 and 2, we have the following. Let α, β be two real numbers and X, Y be two random variables. Then:

$$E[\alpha \cdot X + \beta \cdot Y] = \alpha \cdot E[X] + \beta \cdot E[Y].$$

   - Once again, this can be generalized. For n random variables $X_1, X_2, \ldots, X_n$ and n real numbers $\alpha_1, \alpha_2, \ldots, \alpha_n$:

$$E\left[\sum_{i=1}^{n} \alpha_i \cdot X_i\right] = \sum_{i=1}^{n} \alpha_i \cdot E[X_i].$$

4. Let $g(X)$ be a function of the random variable. Then, the expectation of $g(X)$ is denoted by $E[g(X)]$ and is equal to:

   - for discrete random variable X with sample space S:

   $$E[g(X)] = \sum_{x \in S} g(x) \cdot p(x).$$

   - for continuous random variable X:

   $$E[g(X)] = \int_{-\infty}^{+\infty} g(x) \cdot f(x) dx.$$

   Let us see a couple of examples.

Profit expectation

A company makes \$2,000 if they sell 4 units, \$1,800 if they sell 3 units, \$1,200 if they sell 2 units, lose \$1,000 if they sell 1 unit, and lose \$3,000 if they sell no units. Each event from 0 to 4 customers is equally probable. How much should they expect to make?

$$\begin{aligned} E[g(X)] = \sum_{x=0}^{4} g(x) \cdot p(x) = \\ &= 2000 \cdot \frac{1}{5} + 1800 \cdot \frac{1}{5} + 1200 \cdot \frac{1}{5} - 1000 \cdot \frac{1}{5} - 3000 \cdot \frac{1}{5} = \\ &= \$1000. \end{aligned}$$

Circuit heat

Let X be a continuous random variable measuring the current (in milliamperes, mA) in a wire with pdf $f(x) = 0.05$, for $0 \leq x \leq 20$. The heat produced from the current is given by the function $g(x) = 10 \cdot x$ (with x in milliamperes). What is the mean heat produced by the current?

$$\begin{aligned} E[g(X)] &= \int_{x=0}^{20} g(x) \cdot f(x) \cdot dx = \int_{x=0}^{20} g(x) \cdot f(x) dx = \\ &= \int_{x=0}^{20} 10 \cdot x \cdot 0.05 \cdot dx = \int_{x=0}^{20} 0.5 \cdot x \cdot dx = 100. \end{aligned}$$

Variance

Expectations are important; they are also utterly revealing of a single point of interest. Your decision-making process is bound to be very different if I tell you that the expectation is you will make $1000 in the following two scenarios:

1. You will make $500 or $1500 with probability 50% each;

2. You will lose $3000 or make $5000 with probability 50% each.

While in both cases the expected value is $1000, the second one is much more "spread out" than the first one (where all values that random variable X can take are closer to the expectation).

Variance is a quantity that helps answer the question "how spread out is my distribution?" or "what is the variability of a random variable?" Once again, due to the fact that random variables are random, variances are important and they reveal the "spread" of the probability distribution. A variance of 0 implies that the expectation always comes true.

Definition 4 (Variance) *With the term variance, we imply a measure of the spread of the probability distribution. Intuitively, we may think of the variance of a random variable X as the expected squared deviation of the values that X is allowed to take compared to the expected value of X.*

In mathematical terms:

$$Var\,[X] = E\left[(X - E[X])^2\right] = E\left[X^2\right] - (E\,[X])^2\,.$$

The definition implies that the variance is **always nonnegative**! That is, we always have

$$Var\,[X] \geq 0.$$

The variance is sometimes replaced by the standard deviation.

Definition 5 (Standard deviation) *Standard deviation is also a measure of the spread of a probability distribution. It is represented by $SD\,[X]$ and is related to the variance with the following expression:*

$$SD\,[X] = \sqrt{Var\,[X]}.$$

Unsurprisingly, it is commonly denoted by σ.

We refer to this measure as the standard deviation because it *standardizes* the units of the deviation. Note the following:

- Assume that X is a random variable measured in *units* (e.g., miles, Kelvin, $, etc.).

- Then, $E[X]$ (or μ) is the expectation of random variable X and it is also measured in units.
- On the other hand, $Var[X]$ (or σ^2) is the variance of random variable X and it is measured in units *squared* (e.g., miles^2, Kelvin^2, \$^{2}).
- Contrary to the variance, $SD[X]$ (or σ) is the standard deviation of random variable X and it is measured in units (the same as X, miles, Kelvin, \$).

Now, like we did for expectations, we separate the discussion in discrete and continuous random variables. From now on, whenever we are interested in the standard deviation we may simply take the squared root of the variance.

Discrete random variables

Definition 6 (Variance of a discrete random variable) *Let X be a numerically-valued discrete random variable with sample space S and probability mass function $p(x)$. Then, the variance of X is written as $Var[X]$ and is calculated as:*

$$Var[X] = E\left[(X - E[X])^2\right] = \sum_{x \in S} (x - E[X])^2 \cdot p(x).$$

The variance is commonly written as σ^2.

An unfair die

Consider the same "unfair" die as before with sample space $S = \{1,2,3,4,5,6\}$ and $p(1) = 1/3, p(2) = 1/6, p(3) = 1/6, p(4) = 1/6, p(5) = 1/12, p(6) = 1/12$. What is the variance?

Remember that $E[X] = 2.75$, as we calculated earlier. Then, applying the formula for discrete random variables, we get:

- $Var[X] = (1-2.75)^2 \cdot \frac{1}{3} + (2-2.75)^2 \cdot \frac{1}{6} + (3-2.75)^2 \cdot \frac{1}{6} + (4-2.75)^2 \cdot \frac{1}{6} + (5-2.75)^2 \cdot \frac{1}{12} + (6-2.75)^2 \cdot \frac{1}{12} = \frac{33}{12} = 2.6875$.

Recall that per Definition 4 $Var[X]$ is also equal to $E[X^2] - (E[X])^2$. Hence, we could have answered the question, using a slightly different logic:

An unfair die: second take

Remember that $E[X] = 2.75$, as we calculated earlier. Now, applying the other formula, we get:

- $Var[X] = E[X^2] - (E[X])^2 = E[X^2] - 2.75^2$.

Let us focus on the first quantity ($E[X^2]$). We have a function of a random variable, $g(X) = X^2$. Hence, applying the fourth of the expectation properties, we may compute this as:

- $E[X^2] = \sum_{x=1}^{6} x^2 \cdot p(x) = 1^2 \cdot \frac{1}{3} + 2^2 \cdot \frac{1}{6} + 3^2 \cdot \frac{1}{6} + 4^2 \cdot \frac{1}{6} + 5^2 \cdot \frac{1}{12} + 6^2 \cdot \frac{1}{12} = 10.25$.

Subtracting $2.75^2 = 7.5625$, we get that $Var[X] = 2.6875$, as expected.

What about for a fair die, where each side (1, 2, 3, 4, 5, or 6) are equally probable? What is the variance for this die? Provide a brief explanation why there is a difference in the variance of the two dies.

Continuous random variables

Definition 7 (Variance of a continuous random variable) *Let X be a numerically-valued continuous random variable defined over $(-\infty, +\infty)$ with probability distribution function $f(x)$. Then, the variance of X is written as $Var[X]$ and is calculated as:*

$$Var[X] = E\left[(X - E[X])^2\right] = \int_{-\infty}^{+\infty} (x - E[X])^2 \cdot f(x)dx.$$

The variance is commonly written as σ^2.

Back to the rating system

Earlier, we saw that the expected rating for an employee in the company was 3.77. How about the variance? Remember that the ratings are between 1 and 5 (continuous) and have pdf $f(x) = \frac{3}{124} \cdot x^2$.

We again apply the formula (but for continuous random variables now) and get:

- $Var[X] = \int_{-\infty}^{+\infty} (x - E[X])^2 \cdot f(x)dx = \int_{1}^{5} (x - 3.77)^2 \cdot dx = \int_{1}^{5} (x - 3.77)^2 \cdot dx = \int_{-2.77}^{1.23} y^2 \cdot dy = \frac{y^3}{3}\Big|_{-2.77}^{1.23} = \frac{(1.23)^3}{3} - \frac{(-2.77)^3}{3} = 7.7.$

Like we did earlier, we can again apply the formula that $Var[X] = E[X^2] - (E[X])^2$ and get the same result. This is left as an exercise to the reader.

Earlier, we saw that good employees receive a rating score of 4 or above and the distribution of their scores has pdf $f(x) = \frac{2}{9} \cdot x$ for $4 \leq x \leq 5$. What is the variance of the rating score of a good employee?

Properties of the variance

The variance satisfies the following properties.

1. Let α be a real number (not a random variable). Then:

$$Var[\alpha] = 0.$$

   - In essence, this states that when you know what is going to happen, there is no variance!

2. Let α be a real number and X be a random variable. Then:

$$Var[\alpha \cdot X] = \alpha^2 \cdot Var[X].$$

3. Let X, Y be two **independent** [1] random variables. Then:

$$Var[X + Y] = Var[X] + Var[Y].$$

   - Like with the expectation, this also generalizes to more than two random variables. We then have for n **independent** random variables $X_1, X_2, \ldots, X_n$:

$$Var\left[\sum_{i=1}^{n} X_i\right] = \sum_{i=1}^{n} Var[X_i].$$

[1] We did not need this assumption when we were dealing with expectations!

- Let α, β be real numbers and X be a random variable. Combining 1, 2, and 3 leads to:

$$Var[\alpha \cdot X + \beta] = Var[\alpha \cdot X] + Var[\beta] = \alpha^2 \cdot Var[X].$$

4. We combine 2 and 3 to get the following. Let α, β be two real numbers and X, Y be two independent random variables. Then:

$$Var[\alpha \cdot X + \beta \cdot Y] = \alpha^2 \cdot Var[X] + \beta^2 \cdot Var[Y].$$

- In general, for n independent random variables $X_1, X_2, \ldots, X_n$ and n real numbers $\alpha_1, \alpha_2, \ldots, \alpha_n$:

$$Var\left[\sum_{i=1}^{n} \alpha_i \cdot X_i\right] = \sum_{i=1}^{n} \alpha_i^2 \cdot Var[X_i].$$

Expectation and variance of well-known distributions

In this part, we will turn our focus to the distributions we have already discussed. What is the expected number of successes in n trials? What is the expected number of earthquakes in the next decade? What is the variance of a Gamma distributed random variable? We will both derive and apply these quantities in this coming part.

Bernoulli, binomial, geometric, hypergeometric

Bernoulli distribution Recall that we say X is Bernoulli distributed if it can take two values 0 or 1 (failure or success) with probabilities $q = 1 - p$ and p, respectively. Then, based on the definition of expectation, we have:

$$E[X] = p \cdot 1 + (1 - p) \cdot 0 = p.$$

Similarly, based on the definition of variance, we get:

$$Var[X] = E\left[X^2\right] - (E[X])^2 = p \cdot 1^2 + (1 - p) \cdot 0^2 - p^2 = p \cdot (1 - p).$$

Binomial distribution We again apply the formula from the definition of the binomial distribution with parameters n and p:

$$\begin{aligned}
E[X] &= \sum_{x=0}^{n} x \cdot p(x) = \sum_{x=0}^{n} x \binom{n}{x} p^x (1-p)^{n-x} = \\
&= \sum_{x=0}^{n} x \frac{n!}{x! \cdot (n-x)!} p^x (1-p)^{n-x} = \\
&= \sum_{x=0}^{n} \frac{n \cdot (n-1)!}{(x-1)! \cdot (n-x)!} p \cdot p^{x-1} \cdot (1-p)^{n-x} = \\
&= np \sum_{x=1}^{n} \binom{n-1}{x-1} p^{x-1} (1-p)^{n-x} = \\
&= np \sum_{k=0}^{n-1} \binom{n-1}{k} p^k (1-p)^{n-k-1} = && (k = x-1) \\
&= np \sum_{k=0}^{m} \binom{m}{k} p^k (1-p)^{m-k} = && (m = n-1) \\
&= np
\end{aligned}$$

Why is $\sum_{k=0}^{m} \binom{m}{k} p^k (1-p)^{m-k} = 1$?

Finally, we omit the derivation for the variance, but the end result is that:

$$Var[X] = n \cdot p \cdot (1-p).$$

A certificate program

Students accepted in a certificate program graduate with probability $p = 0.75$. This year, the certificate program has accepted 300 students. How many are expected to successfully finish the program?

Let X be the random variable of the number of students that successfully finish the program. Then:

$$E[X] = n \cdot p = 300 \cdot 0.75 = 225 \text{ students.}$$

Geometric distribution We now have:

$$E[X] = \frac{1}{p}$$

and

$$Var[X] = \frac{1-p}{p^2}.$$

Shooting free throws

A kid learning basketball is shooting free throws with a probability of scoring equal to 25%. What are the expected free throws the kid has to attempt until scoring for the first time?

Let X be the number of free throws shot until the first one is made. X is a geometric random variable with $p = 0.25$, hence:

$$E[X] = \frac{1}{p} = 4 \text{ free throws.}$$

Hypergeometric distribution As a reminder, we have a population of N elements, K of which are successes (and $N - K$ are failures). We pick a sample of size n from the big population of N elements. We, then, have for a random variable X that is following a hypergeometric distribution:

$$E[X] = n \cdot \frac{K}{N}$$

and

$$Var[X] = n \frac{K}{N} \frac{(N-K)}{N} \frac{N-n}{N-1}.$$

Poisson, exponential, and Gamma

In all three distributions, we assume the availability of a rate parameter $\lambda > 0$.

Poisson distribution As a reminder, $p(x) = e^{-\lambda} \cdot \frac{\lambda^x}{x!}$. Let us apply this in the general expectation formula:

$$\begin{aligned} E[X] &= \sum_{x=0}^{\infty} x \cdot p(x) = \sum_{x=0}^{\infty} x e^{-\lambda} \cdot \frac{\lambda^x}{x!} = \\ &= \sum_{x=1}^{\infty} x e^{-\lambda} \cdot \frac{\lambda^x}{x!} = \\ &= \lambda \cdot e^{-\lambda} \sum_{x=1}^{\infty} \frac{\lambda^{x-1}}{(x-1)!} = \\ &= \lambda \cdot e^{-\lambda} \sum_{y=0}^{\infty} \frac{\lambda^y}{y!} = \\ &= \lambda \cdot e^{-\lambda} \cdot e^{\lambda} = \\ &= \lambda. \end{aligned}$$

For the variance, we have:

$$\begin{aligned}
Var[X] &= E\left[X^2\right] - (E[X])^2 = \sum_{x=0}^{\infty} x^2 \cdot p(x) - \lambda^2 = \\
&= \sum_{x=0}^{\infty} x^2 e^{-\lambda} \cdot \frac{\lambda^x}{x!} - \lambda^2 = \\
&= \sum_{x=1}^{\infty} x e^{-\lambda} \cdot \frac{\lambda^x}{x!} - \lambda^2 = \\
&= \lambda \cdot e^{-\lambda} \sum_{x=1}^{\infty} x \cdot \frac{\lambda^{x-1}}{(x-1)!} - \lambda^2 = \\
&= \lambda \cdot e^{-\lambda} \left(\sum_{x=1}^{\infty} \lambda \cdot \frac{\lambda^{x-2}}{(x-2)!} + \sum_{x=1}^{\infty} \frac{\lambda^{x-1}}{(x-1)!} \right) - \lambda^2 = \\
&= \lambda \cdot e^{-\lambda} \left(\sum_{y=0}^{\infty} \lambda \cdot \frac{\lambda^{y}}{y!} + \sum_{y=0}^{\infty} \frac{\lambda^{y}}{y!} \right) - \lambda^2 = \\
&= \lambda \cdot e^{-\lambda} \left(\lambda e^{-\lambda} + e^{-\lambda} \right) - \lambda^2 = \\
&= \lambda^2 + \lambda - \lambda^2 = \lambda.
\end{aligned}$$

Hence, both the expectation and the variance of the Poisson is distribution is λ.

Exponential distribution This is a continuous distribution, hence the derivation of the expectation and the variance slightly change.

$$\begin{aligned}
E[X] &= \int_0^{\infty} x \cdot f(x) \cdot dx = \int_0^{\infty} \lambda x e^{-\lambda x} \cdot dx = \\
&= \frac{1}{\lambda} \cdot \int_0^{\infty} y e^{-y} dy \qquad\qquad (y = \lambda \cdot x \implies dx = dy/\lambda) \\
&= \frac{1}{\lambda} \left(-e^{-y} - y e^{-y} \right) \Big|_0^{\infty} = \frac{1}{\lambda}.
\end{aligned}$$

The variance is

$$Var[X] = \frac{1}{\lambda^2}.$$

Gamma distribution We finish this part with the Gamma distribution.

$$\begin{aligned}
E[X] &= \frac{n}{\lambda}, \\
Var[X] &= \frac{n}{\lambda^2}.
\end{aligned}$$

Chasing cars

A transportation engineer is counting vehicles that are passing through an intersection. They have observed that vehicles pass following a Poisson distribution with rate 1 vehicle every 30 seconds.

- The expected number of vehicles in the next 30 seconds is
$$\lambda = 1.$$
- The expected time until the next vehicle is
$$\frac{1}{\lambda} = \frac{1}{1/30 \text{ seconds}} = 30 \text{ seconds}.$$
- The expected time until the 10th vehicle passes is
$$\frac{10}{\lambda} = \frac{10}{1/30 \text{ seconds}} = 300 \text{ seconds} = 5 \text{ minutes}.$$

Be careful with the rate you are using! In general, given a rate λ in some time unit, then if we are asked to find an expectation in time t, we need to replace λ with $\lambda \cdot t$.

Chasing cars: part 2

A transportation engineer is counting vehicles that are passing through an intersection. They have observed that vehicles pass following a Poisson distribution with rate 1 vehicle every 30 seconds. How many vehicles should they expect to see in 3 hours?

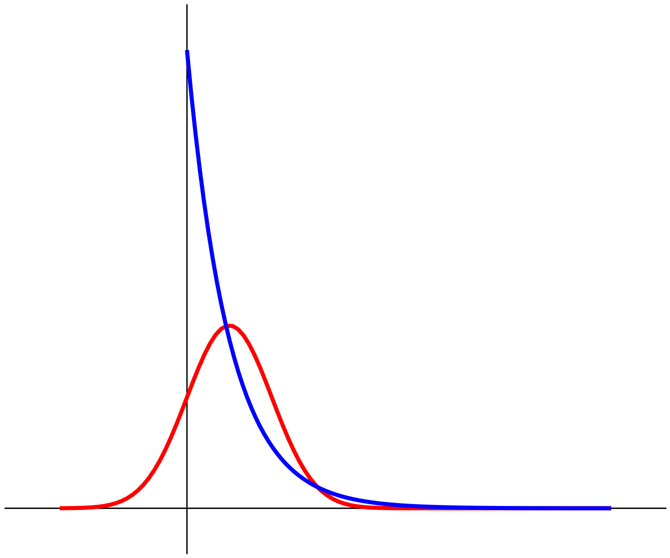
- This is still a Poisson distribution with a rate of 1 every 30 seconds.
- That said, it would be easier to transform the rate into the period asked – 3 hours.
- 3 hours $= 10800$ seconds $\implies \lambda = 360$ vehicles per 3 hours.

Uniform

Recall that there is a discrete and a continuous uniform distribution. We have:

- Discrete between a and b, that is X takes values in $a, a+1, \ldots, b$:
  1. $E[X] = \frac{a+b}{2}$.

Figure 1: An example of how two distributions can have the same mean and variance but yet not be similar at all.



2. $Var[X] = \frac{(b-a+1)^2-1}{12}$.

- Continuous in (a, b):

1. $E[X] = \frac{a+b}{2}$.
2. $Var[X] = \frac{(b-a)^2}{12}$.

Normal

Last, but not least, we see the normal. The good news is that μ and σ^2 are both in the definition of the distribution!

Food for thought

Are two distributions with the same expectation and variance the same distributions? The answer is no: consider for a counterexample an exponential distribution with $\lambda = 0.5$ and a normal distribution $\mathcal{N}(2, 4)$. Their means are both equal to 2 and their variances are both equal to 4, but they are categorically not the same distribution (see Figure 1).

Review

Discrete random variables

Name	**Parameters**	**Values**	**pmf**	**E [X]**	**Var [X]**
Bernoulli	$0 < p < 1$	$\{0, 1\}$	$p(0) = 1 - p$ $p(1) = p$	p	$p(1-p)$
Binomial	$0 < p < 1, n \geq 0$	$\{0, 1, \ldots, n\}$	$p(x) = \binom{n}{x} p^x \cdot (1-p)^{n-x}$	np	$np(1-p)$
Geometric	$0 < p < 1$	$\{1, 2, \ldots\}$	$p(x) = (1-p)^{x-1} \cdot p$	$\frac{1}{p}$	$\frac{1-p}{p^2}$
Hypergeometric	$N, K, n \geq 0$	$\{1, 2, \ldots\}$	$p(x) = \frac{\binom{K}{x}\binom{N-K}{n-x}}{\binom{N}{n}}$	$n\frac{K}{N}$	$n \cdot \frac{K}{N} \cdot \frac{N-K}{N} \cdot \frac{N-n}{N-1}$
Poisson	$\lambda > 0$	$\{0, 1, \ldots\}$	$p(x) = e^{-\lambda} \frac{\lambda^x}{x!}$	λ	λ
Uniform	-	$[a, b]$	$p(x) = \frac{1}{b - a + 1}$	$\frac{a+b}{2}$	$\frac{(b-a+1)^2-1}{12}$

Continuous random variables

Name	**Parameters**	**Values**	**pdf**	**E [X]**	**Var [X]**
Uniform	-	$[a, b]$	$f(x) = \frac{1}{b-a}$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$
Exponential	$\lambda > 0$	$[0, +\infty)$	$f(x) = \lambda \cdot e^{-\lambda x}$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$
Gamma	$\lambda > 0, k > 0$	$[0, +\infty)$	$f(x) = \frac{\lambda^k \cdot x^{k-1} \cdot e^{-\lambda x}}{\Gamma(k)}$	$\frac{n}{\lambda}$	$\frac{n}{\lambda^2}$
Erlang	$\lambda > 0$, integer $k > 0$	$[0, +\infty)$	$f(x) = \frac{\lambda^k \cdot x^{k-1} \cdot e^{-\lambda x}}{(k-1)!}$	$\frac{n}{\lambda}$	$\frac{n}{\lambda^2}$
Normal	μ, σ^2	$(-\infty, +\infty)$	$f(x) = \frac{1}{\sqrt{2\pi} \cdot \sigma} e^{\frac{(x-\mu)^2}{2\sigma^2}}$	μ	σ^2