Expectations

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Lecture 9a

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ISE | Industrial & Enterprise Systems Engineering
GRAINGER COLLEGE OF ENGINEERING

Last time..

- Finished with continuous random variables.
- We discussed:
  - Uniform;
  - Exponential;
  - Gamma and Erlang;
  - Normal.
- Recall that the normal distribution required two parameters, referred to as μ and σ^2...

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- Recall that the normal distribution required two parameters, referred to as μ and σ^2...

Today, we will discuss expectations and variances.

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Parameters of a distribution

DISCRETE

Name	Parameters	Values	pmf
Bernoulli	$0 < p < 1$	$\{0, 1\}$	$p(0) = 1 - p$ $p(1) = p$
Binomial	$0 < p < 1$, $n \geq 0$	$\{0, 1, \ldots, n\}$	$p(x) = \binom{n}{x} p^x \cdot (1-p)^{n-x}$
Geometric	$0 < p < 1$	$\{1, 2, \ldots\}$	$p(x) = (1-p)^{x-1} \cdot p$
Hypergeometric	$N, K, n \geq 0$	$\{1, 2, \ldots\}$	$p(x) = \frac{\binom{K}{x}\binom{N-K}{n-x}}{\binom{N}{n}}$
Poisson	$\lambda > 0$	$\{0, 1, \ldots\}$	$p(x) = e^{-\lambda} \frac{\lambda^x}{x!}$
Uniform	-	$[a, b]$	$p(x) = \frac{1}{b - a + 1}$

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Parameters of a distribution

CONTINUOUS

Name	Parameters	Values	pdf
Uniform	-	$[a, b]$	$f(x) = \frac{1}{b-a}$
Exponential	$\lambda > 0$	$[0, +\infty)$	$f(x) = \lambda \cdot e^{-\lambda x}$
Gamma	$\lambda > 0, k > 0$	$[0, +\infty)$	$f(x) = \frac{\lambda^k \cdot x^{k-1} \cdot e^{-\lambda x}}{\Gamma(k)}$
Erlang	$\lambda > 0$, integer $k > 0$	$[0, +\infty)$	$f(x) = \frac{\lambda^k \cdot x^{k-1} \cdot e^{-\lambda x}}{(k-1)!}$
Normal	μ, σ^2	$(-\infty, +\infty)$	$f(x) = \frac{1}{\sqrt{2\pi} \cdot \sigma} e^{\frac{(x-\mu)^2}{2\sigma^2}}$

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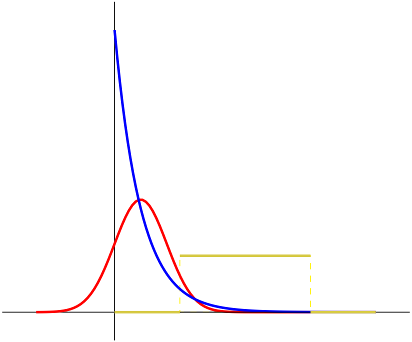
Questions

1. What do the parameters reveal about the shape of the distribution?
2. Knowing the parameters, can we answer questions about what we should *expect* will happen?
3. Knowing the parameters, can we answer questions about *how far from the expectation* we may find ourselves?



Mean, variance, standard deviation

Specifically, in this class we will provide definitions for:

- **Mean** (or **expectation**, or **expected value**) is a measure of the "center" of the probability distribution, usually denoted by $E[X]$ or μ.
- **Variance** is a measure of the variability of the probability distribution, denoted by $Var[X]$ or σ^2.
- **Standard deviation** is another measure of variability, and is defined as the square root of the variance, denoted by $SD[X]$ or σ.
  - It is called "standard" as it standardizes variability to the same unit of the original random variable.

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Expected values

We separate our discussion between expected values for:

- Discrete random variables:
- Continuous random variables:

Expected values

We separate our discussion between expected values for:

- Discrete random variables:

Definition

Let X be a numerically-valued discrete random variable with sample space S and probability mass function $p(x)$. Then, the expected value of X is written as $E[X]$ and is calculated as:

$$E[X] = \sum_{x \in S} x \cdot p(x).$$

The expected value is commonly referred to as the *mean* and is also written as μ.

- Continuous random variables:

Expected values

We separate our discussion between expected values for:

- Discrete random variables:

$$\boxed{E[X] = \sum_{x \in S} x \cdot p(x).}$$

- Continuous random variables:

Expected values

We separate our discussion between expected values for:

- Discrete random variables:

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- Continuous random variables:

Definition

Let X be a numerically-valued real random variable defined over $(-\infty, +\infty)$ and probability density function $f(x)$. Then, the expected value of X is written as $E[X]$ and is calculated as:

$$E[X] = \int_{-\infty}^{+\infty} x \cdot f(x) dx.$$

The expected value is commonly referred to as the *mean* and is also written as μ.

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Recall that:

- $\sum_{x \in S} p(x) = 1.$
- $\int_{-\infty}^{+\infty} f(x) dx = 1.$

How to calculate means

- Discrete random variables: $$E[X] = \sum_{x \in S} x \cdot p(x).$$
- Continuous random variables: $$E[X] = \int_{-\infty}^{+\infty} x \cdot f(x)dx.$$

Example

A (discrete) random variable is distributed with $p(x) = x^2/c, x = 1, 2, 3, 4$.

- *What is the mean value?*
- *What if the random variable is continuous in $[1, 4]$?*

Answer:

$$\sum_{x=1}^{4} x \cdot p(x) = \sum_{x=1}^{4} \frac{x^3}{c} = \frac{100}{c}.$$

$$\int_{x=1}^{4} x \cdot f(x) = \int_{x=1}^{4} \frac{x^3}{c} = \frac{x^4}{4c}\Big|_1^4 = \frac{63.75}{c}.$$

From $\sum_{x=1}^{4} p(x) = 1$, we get that $c = 30$.

From $\int_{x=1}^{4} f(x) = 1$, we get that $c = 21$.

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Means of discrete random variables

- Bernoulli with probability p (assume failure=0 & success=1):

$$E[X] = 1 \cdot p + 0 \cdot (1 - p) = p.$$

- Binomial with parameters p and n:

$$E[X] = n \cdot p.$$

Example

Students accepted in a certificate program graduate with probability $p = 0.75$. This year, the certificate program has accepted 300 students. How many are expected to successfully finish the program?

Answer: Binomial with $n = 300$, $p = 0.75$, hence $\mu = n \cdot p = 225$ students.

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Derivation of mean for binomial

From the definition of expectations for discrete random variables:

$$
\begin{aligned}
E[X] = \sum_x x \cdot p(x) = \sum_{x=0}^{n} x \binom{n}{x} p^x (1-p)^{n-x} = \\
= \sum_{x=0}^{n} x \frac{n!}{x! \cdot (n-x)!} p^x (1-p)^{n-x} = \\
= \sum_{x=0}^{n} \frac{n \cdot (n-1)!}{(x-1)! \cdot (n-x)!} p \cdot p^{x-1} \cdot (1-p)^{n-x} = \\
= np \sum_{x=1}^{n} \binom{n-1}{x-1} p^{x-1} (1-p)^{n-x} = \\
= np \sum_{k=0}^{n-1} \binom{n-1}{k} p^k (1-p)^{n-k-1} = \qquad & (k = x-1) \\
= np \sum_{k=0}^{m} \binom{m}{k} p^k (1-p)^{m-k} = \qquad & (m = n-1) \\
= np.
\end{aligned}
$$

Means of discrete random variables

- Geometric with parameter p:

$$E[X] = \frac{1}{p}.$$

Example

A kid learning basketball is shooting free throws with a probability of scoring equal to 25%. What are the expected free throws the kid has to attempt until scoring for the first time?

Answer: Geometric with $p = 0.25$, hence $\mu = \frac{1}{p} = 4$ free throws.

- Hypergeometric with parameters N, K, n:

$$E[X] = n\frac{K}{N}.$$

Example

A trick-or-treat package contains 100 items, 20 of them are chocolate bars. A kid picks 10 items at random; how many chocolate bars should they expect?

Answer: Hypergeometric with $N = 100$, $K = 20$, $n = 10$, hence $\mu = n\frac{K}{N} = 2$ chocolate bars.

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- Poisson with parameter λ:

$$E[X] = \lambda.$$

Note: when interested in finding the expected number of events given a rate λ during a period t, we can find that as $\boxed{\lambda \cdot t}$.

Example

A transportation engineer has installed a sensor to measure the number of vehicles passing through an intersection. The number of vehicles is Poisson distributed with rate $\lambda = 60/hour$. What is the expected number of vehicles in 1 hour? What is the expected number of vehicles in 3 hours?

Answer: Poisson with rate $\lambda = 60/hour$, hence $\mu = \lambda = 60$ vehicles. When $t = 3$ hours, we should expect $\mu = \lambda \cdot t = 180$ vehicles.

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Means of continuous random variables

- Uniform between α and β:

$$E[X] = \frac{\alpha + \beta}{2}.$$

Example

If the next bus arrives uniformly in the next 10 minutes, then the next bus is expected to arrive in $E[X] = 5$ minutes.

- Normal with parameters μ, σ^2:

$$E[X] = \mu.$$

Example

If grades are normally distributed with $\mathcal{N}(80, 12)$, then the expected grade of a student in the class is $E[X] = 80$.

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- Exponential with rate λ:

$$E[X] = \frac{1}{\lambda}.$$

Example

If cars pass through an intersection with rate $\lambda = 60/hour$, then the next car will pass in $E[X] = \frac{1}{\lambda} = 1$ minute.

- Gamma/Erlang with parameters λ and k:

$$E[X] = \frac{k}{\lambda}.$$

Example

If cars pass through an intersection with rate $\lambda = 60/hour$, then the $k = 30$-th car is expected to pass in $E[X] = \frac{k}{\lambda} = \frac{30}{60/hour} = 0.5$ hours.

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Properties

Let α, β be real numbers and X, Y random variables. Then:

1. $E[\alpha] = \alpha$.
2. $E[\alpha \cdot X] = \alpha \cdot E[X]$.
3. $E[X + Y] = E[X] + E[Y]$.
   - Generalizes to $E\left[\sum_{i=1}^{n} X_i\right] = \sum_{i=1}^{n} E[X_i]$.
4. $E\left[\sum_{i=1}^{n} \alpha_i \cdot X_i\right] = \sum_{i=1}^{n} \alpha_i \cdot E[X_i]$
5. $E[a \cdot X + b] = a \cdot E[X] + b$.

Expected values of functions of random variables ($g(X)$):

- discrete: $E[g(X)] = \sum_{x:p(x)>0} g(x) \cdot p(x)$.
- continuous: $E[g(X)] = \int_{-\infty}^{+\infty} g(x) \cdot f(x)dx$.

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Examples

Example

A company makes \$2,000 if they sell 4 units, \$1,800 if they sell 3 units, \$1,200 if they sell 2 units, lose \$1,000 if they sell 1 unit, and lose \$3,000 if they sell no units. Each event from 0 to 4 customers is equally probable. How much should they expect to make?

Answer: $E[g(X)] = \sum_{x=0}^{4} g(x) \cdot p(x) =$

$2000 \cdot \frac{1}{5} + 1800 \cdot \frac{1}{5} + 1200 \cdot \frac{1}{5} - 1000 \cdot \frac{1}{5} - 3000 \cdot \frac{1}{5} = \$1000.$

Example

Let X be a continuous random variable measuring the current (in milliamperes, mA) in a wire with pdf $f(x) = 0.05$, for $0 \le x \le 20$. The heat produced from the current is given by the function $g(x) = 10 \cdot x$ (with x in milliamperes). What is the mean heat produced by the current?

Answer:

$E[g(X)] = \int_{x=0}^{20} g(x) \cdot f(x)dx = \int_{x=0}^{20} 10 \cdot x \cdot 0.05dx = \int_{x=0}^{20} 0.5xdx = 100.$

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Answer:

$E\left[g(X)\right] = \int_{x=0}^{20} g(x) \cdot f(x) dx = \int_{x=0}^{20} 10 \cdot x \cdot 0.05 dx = \int_{x=0}^{20} 0.5 x dx = 100.$

Examples

Example

A company makes \$2,000 if they sell 4 units, \$1,800 if they sell 3 units, \$1,200 if they sell 2 units, lose \$1,000 if they sell 1 unit, and lose \$3,000 if they sell no units. Each event from 0 to 4 customers is equally probable. How much should they expect to make?

Answer: $E[g(X)] = \sum_{x=0}^{4} g(x) \cdot p(x) =$

$2000 \cdot \frac{1}{5} + 1800 \cdot \frac{1}{5} + 1200 \cdot \frac{1}{5} - 1000 \cdot \frac{1}{5} - 3000 \cdot \frac{1}{5} = \$1000.$

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