# *Lecture 2 Worksheet*

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Every worksheet will work as follows.

1. You will be asked to form a group with other students in the class: you can make this as big or as small as you'd like, but groups of 4-5 work best.
2. Read through the worksheet, discussing any questions with the other members of your group.
   - You can call me at any time for help!
   - I will also be interrupting you for general guidance and announcements at random points during the class time.
3. Answer each question (preferably in the order provided) to the best of your knowledge.
4. While collaboration between students is highly encouraged and expected, each student has to submit their own version.
5. You will have 24 hours (see gradescope) to submit your work.

## *Activity 1: Texas hold'em*

Texas hold'em is a variant of poker (a card game) where each player is given 2 starting cards out of the 52 cards in a deck. There are 13 cards of each suit; and there are four suits: $\heartsuit$, $\diamondsuit$, $\spadesuit$, $\clubsuit$. The 13 cards are from 1 to 10, along with three special cards named $J$, $Q$, $K$.

### *Problem 1*

How many combinations of starting cards are there? For the sake of this problem consider that order does not matter (and hence a starting card setup of $\heartsuit 3 \spadesuit 7$ and $\spadesuit 7 \heartsuit 3$ are one and the same). Also assume that the same numbers of different suits are different setups (and hence $\diamondsuit 5 \heartsuit K$ and $\clubsuit K \heartsuit 5$ are two different starting setups). [1]

Answer to Problem 1.

[1] Translation: you have 52 cards to pick from, and you need to see how many ways you have of picking 2 of them – without any care for the order.

*Problem 2*

We define a pair as two same cards (of different suit). For example, ♡$K$♠$K$ and ♡4♢4 are pairs. Considering again that order does not matter, and that the same numbers of different suits are different setups (and, for example, ♡4♢4 and ♣4♡4 are two different setups), how many starting setups that are pairs are there? [2]

[2] If it helps, consider how many pairs of a single card (say, "7"s) you can create. Then, whatever that number is, you can multiply it by all 13 possible cards.

Answer to Problem 2.

*Problem 3*

Considering your answers in Problems 1 and 2, what is the probability that we are dealt a pair in our 2 starting cards?

Answer to Problem 3.

## *Activity 2: Midterm elections*

### *Problem 4*

A country has two political parties: Party A and Party B. A small town has 45 registered voters. Typically in an election, you have to count all votes, and the party with the most votes wins. That said, the town does not want to invest the resources to count all 45 votes and are contemplating a new system. In the new system, **only 3** registered voters are selected at random and asked to vote.

Assume we have independently polled all voters and we are aware that Party A is set to win with 30 votes compared to 15 votes for Party B. If the town implements this new system, what is the probability that Party B is the one that wins the election? [3]

[3] For a hint, look at the Quality Control example in our notes!

Answer to Problem 4.

*Problem 5*

How does the calculation change if we were to pick 5 voters among all registered ones (instead of 3)?

Answer to Problem 5.

*Problem 6*

What do you observe when comparing your answers to Problem 4 and Problem 5? Which of the two parties would prefer a bigger number of voters go to the polls?

Answer to Problem 6.

## *Activity 3: Deriving the permutations and combinations formulae*

### *Problem 7: From multiplications to permutations*

In the lecture, we saw that the formula for a permutation of $n$ items is $P_n = n!$. Use the multiplication rule to show that this is indeed the case. [4]

Answer to Problem 7.

[4] If it helps, consider the case of assigning $n$ items to $n$ people. How many ways are there to assign the first item? After assigning the first item, how many ways are there to assign the second one? How about the $k$-th one (for $k < n$)?

### *Problem 8: From multiplications to permutations of $r < n$ items*

Similarly, in the lecture we derived the formula for a permutation of $r < n$ items from a total of $n$ items. Again, use the multiplication rule to show that $P_{n,r} = \frac{n!}{(n-r)!}$.

Answer to Problem 8.

*Problem 9: From permutations to combinations*

Recall that a permutation $P_{n,r}$ is an ordered sequence of $r$ items out of $n$ possible items, whereas a combination $C_{n,r}$ is an unordered sequence of the same $r$ items. Use the multiplication rule to show that $P_{n,r} = C_{n,r} \cdot r!$. Then, use that fact to derive the combinations formula.

Answer to Problem 9.

## *Activity 4: Summary of results*

### *Problem 10: Committing to memory*

We have seen several different formulae for counting in this lecture. In your answer, provide one example and write the formula associated with each setup.

Answer to Problem 10.

- **Multiplication**:

| Formula | Small example |
| --- | --- |
| | |

- **Permutation of n items**:

| Formula | Small example |
| --- | --- |
| | |

- **Permutation of r<n items**:

| Formula | Small example |
| --- | --- |
| | |

- **Distinguishable permutations of groups of items**:

| Formula | Small example |
| --- | --- |
| | |

- **Combination of r<n items**:

| Formula | Small example |
| --- | --- |
| | |