Lecture 20 Worksheet

Chrysafis Vogiatzis

Every worksheet will work as follows.

1. You will be entered into a Zoom breakout session with other students in the class.
2. Read through the worksheet, discussing any questions with the other participants in your breakout session.
   - You can call me using the "Ask for help" button.
   - Keep in mind that I will be going through all rooms during the session so it might take me a while to get to you.
3. Answer each question (preferably in the order provided) to the best of your knowledge.
4. While collaboration between students in a breakout session is highly encouraged and expected, each student has to submit their own version.
5. You will have 24 hours (see Compass) to submit your work.

Worksheet 1: Our first confidence intervals

Today we build our first confidence intervals!

Radon (a naturally occurring radioactive gas) is considered safe if found in quantities less than or equal to 4.0 picocuries per liter of air, or 4.0 pCi/L. Radon quantity is assumed to be normally distributed. To measure radon, house inspectors collect samples over the course of a few days.

Problem 1: Estimating the unknown mean of Radon

A house inspector has taken measurements using equipment that measures with a standard deviation of 1 pCi/L over a period of 3 days [1] and has found that the level of Radon was $X_1 = 3.6\text{pCi/L}, X_2 = 4.1\text{pCi/L}, X_3 = 3.4\text{pCi/L}$. What is the method of moments point estimate based on the sample obtained? [2]

Answer to Problem 1.

[1] Throughout the worksheet, we will assume that every day leads to exactly one measurement.

[2] Don't forget! The method of moments estimator for the unknown mean of a normal distribution is just the sample average!

Problem 2: Building a confidence interval

So, the average is below 4pCi/L. But, are you sure it is safe to live in the house without doing any treatment for Radon? What is the 90% confidence interval for the mean Radon quantity in the house? Recall that σ is known and is equal to 1pCi/L.

Answer to Problem 2.

Problem 3: Increasing the number of observations

Assume we are collecting 7 days worth of observations, and the average is still 3.7pCi/L. What is the 95% confidence interval now? Would you say the house is safe with probability 95% based on this new confidence interval?

Answer to Problem 3.

Worksheet 2: One-sided confidence intervals

One could claim that doing a two-sided confidence interval is a little conservative. Shouldn't we still want to live in the house if the radon quantity is below the lower bound of the confidence intervals?

Problem 4: One-sided 90% confidence interval

What is the **one-sided** (upper) 90% confidence interval? [3] Assume we have collected $n = 7$ days worth of observations (so a sample of size $n = 7$) with $\overline{X} = 3.7$pCi/L. Also recall that we know $\sigma = 1pCi/L$.

[3] In mathematical terms, what is the $(-\infty, U]$ confidence interval? Of course, Radon cannot be negative: but this shouldn't change the operations we do.

Answer to Problem 4.

Problem 5: One-sided 95% confidence interval

What is the **one-sided** (upper, again) 95% confidence interval?

Answer to Problem 5.

Problem 6: Standard deviations

What if σ was different? The standard deviation depends on our measurement tools, no? So, an interesting follow-up question would be: what is the maximum value of σ such that the one-sided (upper) 95% confidence bound is below 4 pCi/L?

Answer to Problem 6.

Worksheet 3: Errors

Problem 7: Limiting the error for two-sided intervals

Before buying a house, interested buyers want to be sure within a limit about the Radon quantities. So, they ask the house inspector to verify that. How many days worth of observations should the inspector collect to address their concerns with an estimation error that is less than or equal to 0.5pCi/L for the **two-sided** confidence interval? [4]

[4] Look at Page 15 of your notes. :)

Answer to Problem 7.

Problem 8: Limiting the error for one-sided intervals

Recall though that the customers do not care about an error on the lower side of the spectrum! They only care for the upper bound; so they are more interested in a one-sided, upper 95% confidence interval. Still, though, the inspector wants to address their questions with a number of measurements enough to limit the estimation error less than or equal to 0.5pCi/L. How many days should they collect data for when interested in limiting the error for the one-sided (upper) confidence interval? [5]

[5] Think about the difference between two- and one-sided confidence intervals. Then, think about where that difference appears in the formula you used earlier.

Answer to Problem 8.

This leads us to recognize that for one-sided confidence intervals, limiting the error to below E using the proper number of samples n leads to:

$$n = \left(\frac{z_\alpha \sigma}{E}\right)^2.$$

Worksheet 4: Extensions

Problem 9: Estimating the unknown mean of Radon

What if the standard deviation is unknown? [6] Assume that the sample collected (over the course of $n = 7$ days, so $n = 7$ observations) has lead us to $\overline{X} = 3.7\text{pCi/L}$ with sample standard deviation $s = 0.9\text{pCi/L}$. What are the 95% two-sided and one-sided (upper) confidence intervals now? [7]

[6] Then, we need the sample standard deviation. But using it leads to a non-normal sampling distribution...

[7] Use the t-table provided in the notes.

Answer to Problem 9.

Contrast the results you got with the results when σ was known. What do you observe about the width of the intervals?

Problem 10: Estimating the unknown mean of Radon again

What if Radon quantities are not normal? You may again assume that $\overline{X} = 3.7\text{pCi/L}$ and $s = 0.9\text{pCi/L}$, but now we have this after analyzing $n = 40$ observations (more than a month of measurements). What are the 95% two-sided and one-sided (upper) confidence intervals now?

Answer to Problem 10.