Lecture 21 Worksheet

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Every worksheet will work as follows.

1. You will be entered into a Zoom breakout session with other students in the class.
2. Read through the worksheet, discussing any questions with the other participants in your breakout session.
   - You can call me using the "Ask for help" button.
   - Keep in mind that I will be going through all rooms during the session so it might take me a while to get to you.
3. Answer each question (preferably in the order provided) to the best of your knowledge.
4. While collaboration between students in a breakout session is highly encouraged and expected, each student has to submit their own version.
5. You will have 24 hours (see Compass) to submit your work.

Worksheet 1: Reading the χ^2 table

For variance confidence intervals, we need to use the χ^2 distribution. Due to its lack of symmetry, it can be confusing and painful to find the correct values to use. We begin with such an activity to help us get used to it.

Problem 1: Retrieving values

Answer to Problem 1.

- $\chi^2_{0.025,14} =$
- $\chi^2_{0.05,23} =$
- $\chi^2_{0.995,10} =$

Problem 2: Retrieving values to build variance confidence intervals

What are the χ^2 values you'd need to build the following two-sided confidence intervals? [1]

Answer to Problem 2.

- $\alpha = 0.05$, $n = 10$:
- $\alpha = 0.05$, $n = 15$:
- $\alpha = 0.10$, $n = 20$:

[1] Remember! Since χ^2 is not symmetric, you need to find **two values** for each two-sided confidence interval. You'd need one for a one-sided confidence interval.

Worksheet 2: A soil contamination problem

An engineer is concerned about soil contamination. They pick 15 soil samples and measure the contaminant levels finding that the sample average is $\overline{X} = 13.7$ ppm and the sample standard deviation is $s = 3.15$. You may assume that the soil contamination level is normally distributed with unknown mean and variance.

Problem 3: Back to basics

Construct a two-sided 95%-confidence interval for the unknown mean soil contamination, μ. [2]

[2] Check Lecture 20!

Answer to Problem 3.

Problem 4: A variance confidence interval

Construct a two-sided 95%-confidence interval for the unknown variance of the soil contamination, σ^2.

Answer to Problem 4.

Worksheet 3: One-sided confidence intervals

Like we did last time, we again check how to derive one-sided confidence intervals for the unknown variance.

Problem 5: One-sided 95% confidence interval

Using the data from the previous exercises (that is, $n = 15, s = 3.15$), construct a one-sided upper 95%-confidence interval for the unknown variance of the soil contamination, σ^2. [3]

[3] That is, your variance would be in $[0, U]$, since the variance can never be negative!

Answer to Problem 5.

Worksheet 4: Proportions

Residents of major metropolitan areas in the US were asked whether they agree with the following statement:

> "I consider my self environmentally conscious."

The answers they could give were either a "Yes" or a "No".

Problem 6: Estimating the proportion in Portland, OR

Out of $n = 91$ respondents in Portland, 61 answered Yes. Create a 95% percent confidence interval on the proportion of Portland residents considering themselves environmentally conscious.

Answer to Problem 6.

Problem 7: Estimating the proportion in Philadelphia, PA

The same survey in Philadelphia for $n = 100$ respondents gave that 45 of them agreed with the statement. Create a 95% percent confidence interval on the proportion of Philadelphia residents now considering themselves environmentally conscious.

Answer to Problem 7.

Problem 8: Designing an experiment

Assume we want to perform the same survey in other cities in the United States. What is the smallest sample size we would need in order to estimate the proportion of residents that consider themselves environmentally conscious with a margin of error smaller than 3% (with 95% confidence again)?

Answer to Problem 8.