Lecture 26-27 Worksheet

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Every worksheet will work as follows.

1. You will be entered into a Zoom breakout session with other students in the class.
2. Read through the worksheet, discussing any questions with the other participants in your breakout session.
   - You can call me using the "Ask for help" button.
   - Keep in mind that I will be going through all rooms during the session so it might take me a while to get to you.
3. Answer each question (preferably in the order provided) to the best of your knowledge.
4. While collaboration between students in a breakout session is highly encouraged and expected, each student has to submit their own version.
5. You will have 24 hours (see Compass) to submit your work.

Worksheet 1: Hypotheses for means of normally distributed populations

A tablet advertises that it has an all-day battery. The company claims that their battery will expectedly last 24 hours. Batteries of that capacity are assumed to have lives that are normally distributed with (known) standard deviation equal to 1.25 hours.

Problem 1: Formulating the hypothesis

Formulate a suitable hypothesis to check whether the battery life is equal to 24 hours or less than that. [1]

[1] Pay close attention: is this a two-sided or a one-sided hypothesis test?

Answer to Problem 1.

H_0 :

H_1 :

Problem 2: The hypothesis testing procedure

A random sample of 10 tablets revealed a sample average life of 23.2 hours. Using $\alpha = 0.05$, can you support the claim that the battery life is 24 hours? [2]

[2] Recall that σ is known and equal to 1.25 hours.

Answer to Problem 2.

Problem 3: Practicing P-values

What is the P-value for the observed average from Problem 2? Is this a "surprising" value or is it expected based on your decision in Problem 2?

Answer to Problem 3.

This shouldn't surprise us and we should indeed be expecting that P-value $< \alpha$.

Problem 4: Type II errors

What is the β error associated with the hypothesis test assuming the true mean of the battery is actually 23 hours?

Answer to Problem 4.

Problem 5: Type II errors revisited

Assume that the β error you found in Problem 4 is unacceptable. The company is asking you for a way to improve this to which you recommend "get a bigger sample!" While you are right, and a bigger sample should decrease β, we would still like to keep the sample small enough. What is the smallest sample you should use in order for β to be equal to at most 10%?

Answer to Problem 5.

Problem 6: Unknown variance

Earlier, we made the assumption that the variance was known. What if the variance is unknown? If we collect a sample of $n = 10$ tablets that result in sample average $\overline{X} = 23.2$ hours and sample standard deviation $s = 1.8$ hours explain whether you can now reject or fail to reject the null hypothesis. [3]

[3] Use the same (one-sided!) hypothesis as formulated in Problem 1.

Answer to Problem 6.

Worksheet 2: Hypotheses for means of not normally distributed populations

Let us switch gears and move away from the tablet and battery lifetime world. The emergency department in a local hospital has observed that their average waiting time has historically been 45 minutes. The hospital hired new personnel and trained the previous hires in early 2020 in an effort to improve waiting times. We make the assumption that these times are **not normally distributed**.

Problem 7: Stating the hypotheses

State the null and alternate hypotheses for whether the waiting times *have improved* after the effort.

Answer to Problem 7.

H_0 :

H_1 :

Problem 8: Drawing a conclusion

A sample of $n = 94$ patients were audited after the improvement and the sample average waiting time was 42.1 minutes with a sample standard deviation of 10 minutes. Should you reject or fail to reject the hypothesis based on the proper test statistic (under $\alpha = 0.05$)? What is the corresponding P-value?

Answer to Problem 8.

Problem 9: Back to the normal distribution

Would your answer in Problem 8 be different had you known the times were normally distributed? Do not re-solve the problem; simply explain the differences (if any).

Answer to Problem 9.

Worksheet 3: Hypotheses for variances of normally distributed populations

The time to get served at a bank with three tellers [4] is normally distributed. The bank had already studied the standard deviation of the time to serve a customer and had found it to be equal to 5.6 minutes. The bank went ahead and hired an industrial engineer who recommended a new queuing setup. Instead of people waiting in three different lines (one for each teller), there is one super-queue where people wait for the first available teller.

[4] This is unimportant information for the purposes of this exercise.

Problem 10: Variance testing

After testing this new system, the sample standard deviation for $n = 30$ customers was found to be $\sigma = 3.2$ minutes. Under $\alpha = 0.05$, is there enough evidence to support the statement that the new queue results in different time variance?

Answer to Problem 10.

Problem 11: Variance testing

Once more, with the new super-queue, the sample standard deviation for $n = 30$ customers was found to be $\sigma = 3.2$ minutes. Using $\alpha = 0.05$, is there enough evidence to support the statement that the new queue results in smaller time variance?

Answer to Problem 11.